The Detection of Phase Transitions in the South African Market

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MPhil in Mathematical Finance,
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Declaration

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Philosophy in the University of the Cape Town. It has not been submitted before for any degree or examination in any other University.

Signed by candidate

Michael van Gysen

May 21, 2016

Abstract

This dissertation details the performance of two specific trading strategies which are based on the Johansen-Ledoit-Sornette (JLS) model. Both positive and negative bubbles are modelled as a log-periodic power law (LPPL) ending in a finite time singularity. The stock prices of the constituents of the FTSE/JSE Top40 index are taken as inputs to the JLS model from 3 June 2003 to 31 August 2015. It is shown that for certain time horizons into the past, the JLS based trading strategies significantly outperform random trading strategies. However this result is highly dependent on how far the model looks into the past, and if the model is calibrating to positive or negative bubbles. The lack of research with regards to the "stylized facts" of the JLS model, specifically relating to the time horizon and type of bubble, poses a significant hurdle in correctly identifying a LPPL structure in stock prices. These core features of the JLS model were developed from a number of positive bubbles that built up over many years. The results suggest that these features may not apply over all time horizons, and for both types of bubbles.

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Chapter 1

Introduction

Financial crashes, and their associated bubbles, have generated much research in recent years. This is largely due to the enormous amount of wealth and value that is lost in the days and weeks during a market correction. The two key theories that attempt to explain changes in stock prices are the efficient market hypothesis (EMH) and the rational bubbles view (RBV). Central to these theories is the assumption that a traded asset has a fundamental value, often calculated as the expected present value of an asset's future cashflows.

Defining what constitutes a financial bubble is no easy task and is prone to controversies. Kindleberger and Aliber (2011) define a bubble as being a period in time when the price of a traded asset strongly deviates from its intrinsic value. The problem with this definition, as well as the EMH and the RBV, lies in the method of determining the traded asset's intrinsic value. If the process were simple, the likelihood of a financial bubble occurring would be small. Market participants would not invest in an asset they know to be overvalued.

Johansen and Sornette (1999), Johansen *et al.* (2000b) and Johansen *et al.* (2000a) provided a solution to the problem by defining a financial bubble as being a period in time when the price of a traded asset displays "faster than exponential" (or super-exponential) growth. In other words, the growth rate of the price is accelerating hyperbolically. This is in contrast to the common perception that bubbles are simply characterized by a constant price growth rate, which leads to exponential growth in stock prices. The Johansen-Ledoit-Sornette (JLS) model proposes that during a bubble the price of an asset follows a log periodic power law (LPPL) which results from positive feedbacks.

There are two main classes that drive positive feedback. The first class includes a variety of market practices such as option hedging, insurance portfolio strategies, market bid-offer spreads and trend following, to name but a few. See Sornette *et al.* (2013) for a more comprehensive list and references therein. The second driver of positive feedback results from human behaviour, more specifically cooperative herding and the imitation of traders and investors. In other words, the action of one trader leads another to act the same, which again reinforces that action and so on. Imitation and herding is a prevalent characteristic of human behaviour. In a financial markets setting, when pressed for time, energy and information it is practical to make a decision that imitates others (Zhou and Sornette, 2009).

JLS argue that in the market place there is competition between value investors and noise traders. This results in oscillations that are periodic in the logarithm of the time to a critical point t_c. The observed log-periodicity develops from the alternating positive and negative feedbacks generated by value investors and noise traders. Importantly, the word "critical" has a specific mathematical meaning derived from the study of complex systems. A critical point is defined as the explosion to infinity of a normally well-behaved quantity (Johansen *et al.*, 2000a). In the JLS model, the critical time t_c signifies the end of the bubble and the start of a transition to a new phase – most probably a stock crash.

The JLS model asserts that a financial crash does not result from the arrival of a new piece of information; rather, the phase transition is a result of a small influence on a system which is close to criticality. It is the combination of a stock price whose growth is super-exponential, and displaying oscillations around this growth, which are log-periodic. This results in a price process during a bubble phase being described as a log-periodic (hyperbolic) power law (LPPL).

The JLS model has had high predictive accuracy in the past, and a full list of its results in different markets can be found in Sornette *et al.* (2013). The paper by Zhou and Sornette (2009) is particularly relevant to this dissertation as it contains an application of the JLS model and its modifications in the South African market. The authors were able to identify five stocks (out of a basket of 45 stocks) on the Johannesburg Stock Exchange (JSE) that were in a bubble regime, over the period from January 2003 to May 2006. All five stocks experienced an abrupt drop in mid-June 2006, which was in agreement with the predicted t_c.

This dissertation aims to test whether the JLS model is capable of detecting phase transitions (both bubbles and rebounds) on the JSE, by way of implementing two different trading strategies. It is hoped that the model will be capable of achieving returns which are greater than corresponding random returns.

Chapter 2 derives the JLS model, summarizes a list of "stylized facts" and mentions a few criticisms of the model. An extension of the JLS model to detecting stock price rebounds is also discussed. Chapter 3 describes how the model is calibrated while detailing the two different trading strategies that were implemented. Results are expanded upon in Chapter 4, with Chapter 5 concluding.

Chapter 2

The JLS Model

Before reviewing the existing literature on the log-period power law (LPPL) model, it is important to give context to the area of mathematics from which the model is derived, namely complex systems.

Out-of-equilibrium dynamics sets the foundation when it comes to studying complex systems, such as dynamic phase transitions of a heterogeneous system. It involves many microscopic elements interacting with one another to produce collective dynamics that have macroscopic properties (Sornette, 2009a). Complex systems can be found in areas such as neurobiology, evolution, plate tectonics, earthquakes, cognition and financial time series. It is the large scale collective behaviours of individual components that results in rare and sudden transitions.

Two leading distributions characterize complex systems, namely the Gaussian (normal) distribution and the power law distribution. Sornette (2009a) describes the Gaussian distribution as being a "mild" family of distributions in contrast to the "wild" power law family, which forms the basis of the LPPL model.

A probability density function $P(x)$, exhibits a power law tail if

$$P(x) \propto \frac{C_\mu}{x^{1+\mu}}, \qquad \text{for large } x, \tag{2.1}$$

where $\mu > 0$.

The scale factor C_μ for power laws has a similar interpretation as the variance in Gaussian distributions. More importantly, power laws display symmetry of scale invariance, which means that for any real number λ, there exists a real number γ such that

$$P(x) = \gamma P(\lambda x), \qquad \forall x, \tag{2.2}$$

where $\gamma = \lambda^{1+\mu}$. It is crucial to stress that most empirical distributions only display a power law-like shape over a finite range of event sizes, either bounded below and above (Malcai *et al.*, 1997), or above a lower threshold (Mandelbrot, 1983), i.e. only in the tail of the observed distribution .

The study of complex phenomena suggests that power laws emerge close to critical points (Sornette, 2009b). A critical point is a bifurcation point separating two different phases or regimes of a dynamic system. These critical points will be the focus when modelling financial bubbles.

2.1 Derivation of the JLS Model

The JLS model was developed by Johansen and Sornette (1999), Johansen *et al.* (2000b) and Johansen *et al.* (2000a). It is based on the rational expectation setting found in Blanchard and Watson (1982) which states that the observed price p_o of an asset can be written as:

$$p_o = p^* + p, \tag{2.3}$$

where p^* is the fundamental value and p is the bubble component. The focus of the JLS model is on the dynamics of the bubble component, which is independent of the fundamental value p^*.

The model assumes that the bubble component p has a stochastic differential equation of the form:

$$\frac{dp}{p(t)} = \mu(t)dt + \sigma(t)dW - \kappa dj, \tag{2.4}$$

where $\mu(t)$ is the drift component, W is a standard Wiener process, and j is a discontinuous jump with $j = 0$ prior to the crash, and $j = 1$ after the crash. κ represents the amplitude of loss associated with the crash. A jump of j by one unit results after each successive jump, with the dynamics of the jumps being governed by a crash hazard rate $h(t)$ defined as:

$$h(t) \approx B'(t_c - t)^{m-1} + C'(t_c - t)^{m-1} \cos\left[\omega \log(t_c - t) - \phi'\right], \tag{2.5}$$

where B', C', ω and ϕ' are real numbers.

This hazard rate describes the interactions between a network of traders that display herding behaviour. Its derivation is non-trivial, using applications from two-dimensional Ising models (Onsager, 1944) and hierarchical diamond lattices (Derrida *et al.*, 1983). The interested reader can find a thorough explanation in Johansen *et al.* (2000a). The reason why equation (2.5) is an approximation is because a Taylor expansion is used in its derivation. For the purpose of this dissertation one should interpret $h(t)dt$ as being the probability of a crash occurring, over a small interval of time dt, conditional on the fact that the crash has not yet happened. Therefore

$$\mathbb{E}^{\mathbb{Q}}[dj|\mathcal{F}_t] = 1 \times h(t)dt + 0 \times (1 - h(t)dt) = h(t)dt, \tag{2.6}$$

where the superscript $\mathbb{Q}$ indicates that the expectation is taken under the risk-neutral measure, conditional on the filtration up to time t.

A key component of equation (2.5) is the hyperbolic power law growth, which ends in a finite-time singularity t_c. This expresses the positive feedbacks resulting from technical and behavioural mechanisms. According to Sornette *et al.* (2013) the cosine part of equation (2.5) models the existence of accelerating panic that arises during the growth of a bubble. It may also represent inertia in the upward moving price of the asset, which is common during financial bubbles.

Absence of arbitrage implies that $\mathbb{E}^{\mathbb{Q}}[dp|\mathcal{F}_t] = 0$. The expectation of equation (2.4) then shows that,

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}}[dp|\mathcal{F}_t] &= \mu(t)p(t)dt + \sigma(t)p(t)\mathbb{E}^{\mathbb{Q}}[dW|\mathcal{F}_t] - \kappa p(t)\mathbb{E}^{\mathbb{Q}}[dj|\mathcal{F}_t] \\
0 &= \mu(t)p(t)dt + \sigma(t)p(t)(0) - \kappa p(t)(h(t)dt) \\
\mu(t) &= \kappa h(t). \qquad (2.7)
\end{aligned}$$

This indicates that the drift of the SDE in equation (2.4) is a function of the crash hazard rate $h(t)$. In other words, the higher the probability of a crash, the higher the return of the asset needs to be, in order to compensate an investor for taking on more risk. This is meant in a risk-neutral sense as the model assumes the no-arbitrage condition. There is also a small probability that the change in regime will not lead to a crash.

Now, conditional on the fact that no crash has occurred, equation (2.4) can be rewritten as

$$\frac{dp}{p(t)} = \mu(t)dt + \sigma(t)dW = \kappa h(t)dt + \sigma(t)dW. \qquad (2.8)$$

Its conditional expectation leads to

$$\mathbb{E}_t\left[\frac{dp}{p}\right] = \kappa h(t)dt. \qquad (2.9)$$

If we substitute expression (2.5) into (2.9) and integrate it yields

$$\ln \mathbb{E}[p(t)] \approx A + B(t_c - t)^m + C(t_c - t)^m \cos\left(\omega \ln\left(t_c - t\right) - \phi\right), \qquad (2.10)$$

where $A = \ln p(c)$ is the log price at t_c, $B = -\kappa B'/m$, $C = -\kappa C'/\sqrt{m^2 + \omega^2}$ is the amplitude of the oscillations and $0 < \phi < 2\pi$ is a phase parameter. Note that the ϕ is a result of the integration with $\phi \neq \phi'$. The values B, m and ω will be discussed in more detail in section 2.2.

It is important to stress that in expression (2.10), t_c is not the exact time of the crash. The crash could happen at any time before t_c, however this is very unlikely. In the context of the JLS model, one should view the critical time t_c as the most probable end of the bubble regime. The JLS model does not specify what happens after time t_c.

2.2 LPPL Stylized Facts

Most of the elements in equation (2.10) have an important economic interpretation. The exponent for instance captures the positive feedback between traders which leads to a super-exponential price growth. It should also be in the range $[0, 1]$. If $m < 0$, then the price could diverge in a finite amount of time which is unrealistic. Since the integral of the hazard rate $h(t)$ up to $t = t_c$ represents the probability of a crash, it cannot be greater than 1, imposing the upper bound $m \leq 1$. The JLS framework also requires that $h(t)$ accelerates with time, which implies that $B' > 0$ and $m < 1$. Hence $B < 0$ since $m > 0$ by the above reasoning.

Sornette *et al.* (2001), Lin *et al.* (2009) and Johansen and Sornette (2010) have found that for over thirty financial bubbles, the values calibrated for m and ω have been remarkably consistent. Specifically, Johansen and Sornette (2010) have found that m and ω have approximate Gaussian distributions with mean and standard deviations:

$$m \approx 0.33 \pm 0.19$$
$$\omega \approx 6.35 \pm 1.55.$$

Filimonov and Sornette (2013) argue that in order to avoid type I errors (rejecting the LPPL hypothesis when it is in fact true) one should broaden the restrictions on m and ω as follows:

$$0.1 \leq m \leq 0.9$$
$$6 \leq \omega \leq 13.$$

The lower and upper bounds on ω ensure that the log-periodic oscillations are neither too frequent, thus fitting the random noise in the data, nor too rare that they contribute to the trend.

van Bothmer and Meister (2003) derived a more modern filter for financial bubbles which restricts the crash rate from becoming negative. They suggest that

$$b := -Bm - |C|\sqrt{m^2 + \omega^2} \geq 0. \tag{2.11}$$

The above restrictions on B, m, ω and b are seen as being the "stylized features of LPPL" (Filimonov and Sornette, 2013).

It is argued in Zhou and Sornette (2009) that it is unwise to filter for log-periodicity based solely on ω, due to there being other peaks on the harmonics of ω. They complement the determination of log-periodicity by using the number of oscillations defined as:

$$N_{osc} = \frac{\omega}{2\pi} \ln \left| \frac{t_c - t_{\text{first}}}{t_c - t_{\text{last}}} \right|, \tag{2.12}$$

where $[t_{\text{first}}, t_{\text{last}}]$ is the interval used when fitting the LPPL model.

In Zhou and Sornette (2002) they show that for most types of noises, as soon as the number (N_{osc}) of oscillations is 3 or more one can reject the hypothesis that the observed log-periodicity results from noise, at a confidence level higher than 95%. In other words when $N_{osc} > 3$, it is highly unlikely that the oscillatory behaviour is due to random noise. This constraint is added to the list of stylized features when calibrating the LPPL model.

2.3 Criticisms of the LPPL Model

Johansen *et al.* (2000a) develop theory related to the rational expectation of bubbles, which simply assumes that the bubble component p in equation (2.3) is a martingale, i.e.

$$\mathbb{E}[p(t)|\mathcal{F}_s] = p(s) \qquad \forall\, t > s. \tag{2.13}$$

However Feigenbaum *et al.* (2001) suggests that this theory is not robust when it comes to the general formulation of risk aversion. This is because equation (2.10) requires that the expectation of the price process, conditional on no crash having yet occurred, must oscillate log-periodically. Feigenbaum *et al.* (2001) also find that when looking at 1987 stock market crash the log-periodic component does appear when looking at all the data prior to the crash. However this feature disappears when removing the last year of data. Sornette *et al.* (2001) stress that in analysing a critical point such as a crash, it is naïve to remove data which is closest to the critical point, as this is the most important part of the time series.

Bree and Joseph (2010) suggest relaxing the constraint $0 < m < 1$, since those fits that have an exponent that lies outside the range $(0, 1)$ give evidence that there is no super-exponential bubble. Sornette *et al.* (2013) find merit in this reasoning, however they mention that the highly non-linear nature of the LPPL model makes the selection of best fits unreliable in some instances. When calibrating, the constraint that $m \in (0, 1)$ should be kept in order to make sure the calibrated values "pass the financial conditions of good sense".

Some criticisms in recent years warrant clarity with regards to the type of bubbles that can be detected by the JLS model. One should understand that a crash can be endogenous or exogenous. Specifically, the JLS model is designed to detect endogenous crashes, which are preceded by bubbles generated by positive feedback mechanisms such as imitation and herding. Exogenous crashes are fundamentally unpredictable.

Of the 49 outliers of financial drawdowns found in Johansen and Sornette (2010), 22 were attributable to exogenous events, 25 were classified as endogenous crashes preceded by speculative bubbles, and 2 were part of the Japanese anti-bubble. When looking at world market indices only, of the 31 outliers 10 were exogenous, 19 endogenous and 2 were part of the Japanese anti-bubble.

2.4 An Extension to Rebounds

In chapter 1 a financial bubble was defined as being a period in time when the price of a traded asset displays super-exponential growth as a result of positive feedbacks. Yan *et al.* (2012) argue that positive feedbacks can also lead to super-exponential negative growth of the stock price. They refer to these regimes as being "negative bubbles", as opposed to the "positive bubbles" mentioned in section 2.1. In a negative bubble, positive feedbacks reveal the collective behaviour of traders shorting the market, and the subsequent panic. The inverse relationship between positive and negative bubbles is easily seen when one considers currency exchange rates. If the dollar/pound exchange rate is increasing super-exponentially, then the pound/dollar exchange rate will be decreasing super-exponentially.

The JLS model can be adapted to negative bubbles by making the drift rate $\mu(t)$ and crash amplitude κ negative in equation (2.4). The crash hazard rate becomes the rally hazard rate, and is interpreted as being the probability of a rebound occurring over a small interval of time dt, conditional on no rebound having occurred yet. Since $\mu(t) = \kappa h(t) < 0$, the higher the probability of a rebound, the greater the loss investors are willing to bear, in order to gain from the impending rebound.

Equation (2.8) still holds, however the following two inequalities change:

$$B > 0, \qquad b < 0. \tag{2.14}$$

It should be noted that this dissertation will apply the same LPPL stylized facts for negative bubbles and positive bubbles. This is largely due to the sparse amount of literature with regards to applying the JLS model to negative bubbles. Sadly, a thorough evaluation of the LPPL structures that are found in rebounds has not yet been conducted.

Chapter 3

Calibration and Application of the LPPL Model on the Johannesburg Stock Exchange

3.1 Calibration of the JLS Model

Calibration involves the fitting of equation (2.10) to an observed price series $p(t)$ during a time window $t \in [t_{\text{start}}, t_{\text{end}}]$, in order to determine the parameters t_c, m, ω, ϕ, A, B and C. This may be performed by minimizing the sum of the squared residuals:

$$\begin{aligned} &S(t_c, m, \omega, \phi, A, B, C) \\ &= \sum_{i=1}^{N} \left[\ln p(t_i) - A - B(t_c - t_i)^m - C(t_c - t_i)^m \cos\left(\omega \ln\left(t_c - t_i\right) - \phi\right)\right]^2, \end{aligned} \tag{3.1}$$

where N is the number of observed prices in the period $[t_{\text{start}}, t_{\text{end}}]$, with $t_1 = t_{\text{start}}$ and $t_N = t_{\text{end}}$.

The cost function S is highly nonlinear, and due to the presence of many local minima, it is a non-trivial task to minimize. Johansen *et al.* (2000a) were able to reduce the complexity of the optimization by slaving the linear parameters A, B, C to the nonlinear parameters t_c, m, ω, ϕ. They showed that

$$\min_{t_c,m,\omega,\phi,A,B,C} S(t_c, m, \omega, \phi, A, B, C) \equiv \min_{t_c,m,\omega,\phi} S_1(t_c, m, \omega, \phi),$$

where

$$S_1(t_c, m, \omega, \phi) = \min_{A,B,C} S(t_c, m, \omega, \phi, A, B, C). \tag{3.2}$$

The optimization in (3.2) can then be rewritten as:

$$\begin{aligned}\left\{\hat{A}, \hat{B}, \hat{C}\right\} &= \arg\min_{A,B,C} S(t_c, m, \omega, \phi, A, B, C) \\ &= \arg\min_{A,B,C} \sum_{i=1}^{N} \left[y_i - A - Bf_i - Cg_i\right], \end{aligned} \tag{3.3}$$

where $y_i = \ln p(t_i)$, $f_i = (t_c - t_i)^m$, $g_i = (t_c - t_i)^m \cos\left(\omega \ln\left(t_c - t_i\right) - \phi\right)$.

Being linear in the variables A, B, C, the optimization in (3.3) has one unique solution which leads to the matrix equation

$$\begin{pmatrix} N & \sum f_i & \sum g_i \\ \sum f_i & \sum f_i^2 & \sum f_i g_i \\ \sum g_i & \sum f_i g_i & \sum g_i^2 \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{C} \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum y_i f_i \\ \sum y_i g_i \end{pmatrix}, \tag{3.4}$$

and can be solved using an LU decomposition (Turing, 1948).

The remaining variables t_c, m, ω, ϕ are then found by solving the nonlinear optimization problem

$$\left\{\hat{t}_c, \hat{m}, \hat{\omega}, \hat{\phi}\right\} = \arg\min_{t_c, m, \omega, \phi} S_1(t_c, m, \omega, \phi). \tag{3.5}$$

The calibration problem has been made simpler by reducing the number of parameters from 7 to 4. However the minimization in (3.5) is still in a four dimensional space, and contains multiple minima.

Filimonov and Sornette (2013) sought to improve the calibration scheme by reducing the optimization problem from a four dimensional space, to a 3 dimensional space. They used the identity, $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$, to expand (2.10) as follows:

$$\begin{aligned}\ln \mathbb{E}[p(t)] \approx\ & A + B(t_c - t)^m + C(t_c - t)^m \cos\left(\omega \ln\left(t_c - t\right)\right) \cos\phi \\ & + C(t_c - t)^m \sin\left(\omega \ln\left(t_c - t\right)\right) \sin\phi \\ =\ & A + B(t_c - t)^m + C_1(t_c - t)^m \cos\left(\omega \ln\left(t_c - t\right)\right) \\ & + C_2(t_c - t)^m \sin\left(\omega \ln\left(t_c - t\right)\right), \end{aligned}$$

where $C_1 = C\cos\phi$ and $C_2 = C\sin\phi$.

The LPPL function now has 3 nonlinear (t_c, m, ω) and 4 linear (A, B, C_1, C_2) parameters - as opposed to the 4 nonlinear and 3 linear parameters in equation (2.10).

We now seek to minimize the cost function

$$
\begin{aligned}
&F(t_c, m, \omega, \phi, A, B, C_1, C_2) \\
&= \sum_{i=1}^{N} [\ln p(t_i) - A - B(t_c - t_i)^m - C_1 (t_c - t_i)^m \cos(\omega \ln(t_c - t_i)) \\
&- C_2 (t_c - t_i)^m \sin(\omega \ln(t_c - t_i))]^2.
\end{aligned} \tag{3.6}
$$

In a similar fashion to the above we slave the 4 linear parameters to the 3 non-linear parameters to obtain:

$$
\{\hat{t_c}, \hat{m}, \hat{\omega}\} = \text{arg} \min_{t_c, m, \omega} F_1(t_c, m, \omega), \tag{3.7}
$$

where

$$
F_1(t_c, m, \omega) = \min_{A, B, C_1, C_2} F(t_c, m, \omega, \phi, A, B, C_1, C_2). \tag{3.8}
$$

The optimization problem

$$
\left\{\hat{A}, \hat{B}, \hat{C_1}, \hat{C_2}\right\} = \text{arg} \min_{A, B, C_1, C_2} F(t_c, m, \omega, \phi, A, B, C_1, C_2), \tag{3.9}
$$

has a unique solution which can be solved by using an LU decomposition in the following matrix equation:

$$
\begin{pmatrix}
N & \sum f_i & \sum g_i & \sum h_i \\
\sum f_i & \sum f_i^2 & \sum f_i g_i & \sum f_i h_i \\
\sum g_i & \sum f_i g_i & \sum g_i^2 & \sum g_i h_i \\
\sum h_i & \sum f_i h_i & \sum g_i h_i & \sum h_i^2
\end{pmatrix}
\begin{pmatrix}
\hat{A} \\ \hat{B} \\ \hat{C}_1 \\ \hat{C}_2
\end{pmatrix}
=
\begin{pmatrix}
\sum y_i \\ \sum y_i f_i \\ \sum y_i g_i \\ \sum y_i h_i
\end{pmatrix}, \tag{3.10}
$$

where $y_i = \ln p(t_i)$, $f_i = (t_c - t_i)^m$, $g_i = (t_c - t_i)^m \cos(\omega \ln(t_c - t_i))$,
$h_i = (t_c - t_i)^m \sin(\omega \ln(t_c - t_i))$.

Filimonov and Sornette (2013) provide a thorough discussion of the improvements proposed by this new method. Importantly, the dimensionality of the optimization problem is now reduced, with the number of local minima also being reduced. This improves the ability of current search algorithms to find the true minimum.

In attempting to solve the optimization problems in (3.5) and (3.7), most approaches use the taboo search devised by Cvijovic and Klinowski (1995). This algorithm improves the local search method by labelling a visited area as "taboo", restricting the algorithm from returning to previously explored local minimum solutions. Most researches then follow the suggestion of Johansen and Sornette (1999) in using the 10 best solutions of the taboo search as initial conditions for a local Levenberg-Marquardt nonlinear least squares algorithm (Levenberg, 1944), (Marquardt, 1963).

This dissertation used a number of different algorithms that are part of the Global Optimization Toolbox developed in MATLAB (matrix laboratory) (MATLAB, 2015). The specific algorithms that were tested are as follows:

- Global Search – uses a gradient based method *fmincon* to return local and global minima. A description of the algorithm can be found in (Ugray *et al.*, 2007);
- Genetic Algorithm (Holland, 1975) – mimics the principles of biological evolution to determine the population of individual search points;
- Pattern Search – uses a minimal and maximal positive basis pattern in the gradient when searching for an optimal point;
- Simulated Annealing (Kirkpatrick, 1984) - devises a probabilistic search algorithm to improve the current minimum by reducing the search area;
- MultiStart - acquires uniformly distributed start points within certain bounds and implements *lsqnonlin* to find the global minimum;
- Particle Swarm (Eberhart *et al.*, 1995) - creates initial particle locations which update iteratively based on certain criteria which repeats until reaching some stopping criterion.

In order to choose which optimisation technique is the most appropriate a simulation study was conducted. It was found that the Global Search (GS) algorithm was the most accurate, followed by the Particle Swarm (PS) algorithm. However on average, PS was more than 10 times faster than GS. As will be discussed in section 3.3, more than 1,800,000 calibrations will need to be run on market data. Thus, due to practicality and time constraints, the PS algorithm is used in this dissertation for all subsequent calibrations. It should be noted that GS had only marginally better accuracy than PS.

In order to ensure that the PS algorithm was calibrating correctly in MATLAB, the model was tested on the same synthetic data found in Sornette *et al.* (2013). Specifically, a log-periodic power law (LPPL) time series was generated for 240 days with input variables, $A = 10$, $B = -0.1$, $C = 0.02$, $m = 0.7$, $\omega = 10$, $\phi = 1$, $t_0 = 0$, $t_n = 240$, $t_c = 300$. Two different kinds of noise were then added to the LPPL time series, namely:

- Gaussian noise with zero mean and standard deviation equal to 5% of the largest log-price of the 240 observations.

- Student t noise with 4 degrees of freedom, zero mean, and standard deviation equal to 5% of the largest log-price of the 240 observations.

Sornette *et al.* (2013) calibrated the JLS model to 200 different noise inclusive LPPL time series (using taboo search), and recorded the mean and standard deviation for the 200 calibrated variables: $\hat{t_c}, \hat{m}, \hat{\omega}$. However this mean and standard deviation is dependent on the random numbers used to generate the Gaussian and Student t noise. One could easily generate "nice" random numbers that lead to superior calibrations. So in order to compare the results of the Particle Swarm (PS) algorithm, 100 separate trials of 200 different noise inclusive LPPL time series were generated. The 25th, 50th and 75th percentiles were recorded, as well as the mean and standard deviation of $\hat{t_c}, \hat{m}, \hat{\omega}$.

Table 3.1 and 3.2 detail the comparison between taboo search and the particle swarm (PS) algorithm for Gaussian and Student t noise. The tables display the values found in Sornette *et al.* (2013), the 25th, 50th and 75th percentiles of the 100 trials, as well as the mean and standard deviation (in brackets) for each of the calibrated parameters.

Tab. 3.1: Calibration to Gaussian distributed noise.

	Reference	Sornette *et al.* (2013)	25%	50%	75%	PS
t_c	300	296.07 (20.44)	300.71	301.55	302.50	301.60 (18.67)
m	0.7	0.74 (0.15)	0.68	0.69	0.70	0.69 (0.19)
ω	10	9.75 (1.43)	10.03	10.08	10.16	10.09 (1.29)

Tab. 3.2: Calibration to Student t distributed noise.

	Reference	Sornette *et al.* (2013)	25%	50%	75%	PS
t_c	300	296.07 (20.44)	298.03	298.96	300.40	299.16 (23.54)
m	0.7	0.74 (0.15)	0.65	0.66	0.67	0.66 (0.24)
ω	10	9.75 (1.43)	9.85	9.91	10.02	9.93 (1.62)

The above results indicate that the particle swarm algorithm is able to calibrate correctly to the noise-adjusted LPPL time series. For Gaussian noise, the PS calibrated values are much closer to the true variables, when compared to taboo search. The same can be said for the Student t distributed noise, although the values for m are not as accurate.

3.2 Data

The data used as an input to the calibration is the daily stock price of all constituents of the FTSE/JSE Top40 index from 3 June 2003 to 31 August 2015. The price data was adjusted for dividends and corporate actions such as unbundling, spinoffs and stock splits. If a stock was removed from the index it was no longer considered as an investing option. Note that although the index is named the Top40, it usually contains more than 40 stocks.

3.3 Trading Strategies

The aim of this dissertation is not to test whether the JLS model is able to deliver superior returns on the Johannesburg Stock Exchange (JSE). This dissertation seeks to compare JLS based trading strategies to random trading strategies. If the JLS trading strategies outperform the random strategies, then one may conclude that the market has some LPPL structure. The trading strategies assume that one can easily buy or short sell any individual stock in the FTSE/JSE Top40 index, with no transaction costs or borrowing costs.

The JLS model is calibrated for each stock in the index, on each day from 3 June 2003 to 31 August 2015 (3063 trading days). The calibrations are done on fifteen, 100 day intervals into the past, i.e. for every interval $[t - \Delta, t]$ with $\Delta \in \{100, 200, ..., 1500\}$. Assuming the index contains exactly 40 stocks, this requires a total of $40 \times 3063 \times 15 = 1\ 837\ 800$ calibrations. As discussed earlier, this extremely high number of calibrations is the primary reason for using the faster Particle Swarm algorithm over Global Search. The reason for using fifteen increasing intervals into the past is because one cannot know, a priori, over what horizon a positive or negative bubble is likely to form.

Due to the highly non-linear nature of the JLS model, it is possible that a calibration yields parameters that meet the stylized facts associated with a LPPL, yet the price process is actually not a LPPL – although this is quite rare. To limit the number of false positives, the price process is only classified as being in a negative/positive bubble once $N_{\text{consec}} \geq (2\% \times \Delta)$. Where N_{consec} is the number of consecutive calibrations that meet the stylized facts criteria of the JLS model. For example, when looking back 200 trading days into the past, 4 or more consecutive calibrations will have to meet the stylized facts before one can implement one of the two trading strategies mentioned below. Clearly in the case of a positive bubble the hypothetical trader will short sell the stock, while for a negative bubble they will buy the asset.

3.3.1 Trading Strategy 1 (TS1)

The first trading strategy is based on the most important variable in the JLS model, the critical point t_c. This future date marks the most likely time for the stock price to crash or rebound. For this strategy, once a stock is classified as being in a positive/negative bubble, the hypothetical trader waits until the t_c date before buying (negative bubble) or short selling (positive bubble) the stock. If in the time between the bubble signal and the t_c date, a new updated t_c date is found, this new t_c becomes the time to long or short. The trader will then remain invested in the stock until either,

$$\bar{r} > S_{\text{gain}} \quad \text{or} \quad \bar{r} < S_{\text{loss}},$$

where $\bar{r}$ is the return on the long/short position, and

$$S_{\text{gain}} = S_{\text{loss}} \in \{2.5\%, 5\%, 7.5\%, 10\%\}$$

is the maximum positive and negative return. Note that only symmetric values of the stop-loss S_{loss} and stop-gain S_{gain} were considered, i.e. S_{gain} always equals S_{loss} so that no risk preference is assumed for the trader.

This trading strategy is then compared to a random strategy which is identical in every way, except that the choice of stock is randomized. The random strategy will stay invested until either the stop-loss or stop-gain is reached, and will buy or short-sell the asset on the exact same day. For the period 3 June 2003 to 31 August 2015 all trades under TS1 are recorded and a number of summary statistics calculated - see Chapter 4. In total 50 000 random trading strategies were used as the comparison proxy, where each random strategy has the same number of trades in the investment period.

The benefit of this strategy is its ability to reveal whether the JLS model has any power in selecting stocks that are in a bubble. Its drawback is its inability to test whether the JLS model can correctly predict the t_c date. This is remedied in trading strategy 2.

3.3.2 Trading Strategy 2 (TS2)

For this trading scheme one short sells (buys) the underlying as soon as the JLS model indicates that it is in a positive (negative) bubble. The asset is then held until

$$t_c + t_{\text{plus}} \times \Delta,$$

where $t_{\text{plus}} \in \{0\%, 5\%, 10\%, 15\%\}$ and $\Delta \in \{100, 200, ..., 1500\}$. If in the time between the bubble signal and the t_c date, a new updated t_c date is found, the trader stays invested until this new t_c date.

The random trading strategy is identical, however now the t_c date is scrambled – the relevant stock remains the same. Once again 50 000 random trading strategies were used as the comparison proxy, where each random strategy has the same number of trades in the investment period as TS2. As mentioned before, the benefit of this approach is its ability to determine whether the JLS model can correctly predict the t_c date, given that it already believes the stock is in a bubble. The disadvantage of this strategy is that it cannot reveal whether the JLS model accurately determines if a stock is in a bubble.

Chapter 4

Comparing JLS Based Trading Strategies to Random Strategies

As there are calibrations for both positive and negative bubbles with four different choices for $S_{\text{gain}} = S_{\text{loss}} \in \{2.5\%, 5\%, 7.5\%, 10\%\}$, it leads to eight different sub-strategies for trading strategy 1 (TS1). The same can be said for trading strategy 2 (TS2) as there are four different choices for $t_{\text{plus}} \in \{0\%, 5\%, 10\%, 15\%\}$, and two different bubbles. For ease of exposition only the results of the best sub-strategies for positive and negative bubbles will be displayed in this chapter. All other results for strategy 1 can be found in Appendix A, and Appendix B for strategy 2.

4.1 Trading Strategy 1

4.1.1 Positive Bubbles

Table 4.1 details the summary statistics for TS1 when $S_{\text{gain}} = S_{\text{loss}} = 5\%$ for positive bubbles. The first column shows how far into the past the model was looking when calibrating the JLS model. Column 2 details how many trades were executed by TS1 between 2002 and 2015. The next two columns show the percentage of trade returns ($\bar{r}$) over the investment period that reached the positive stop gain, S_{gain}. To be clear, for each random trial the percentage of trades that reached S_{gain} was recorded. Column 4 is then an average of these 50 000 percentages. The 25th, 50th and 75th percentiles of these 50 000 random percentages can be found in the last three columns. Finally column 5 shows how many more JLS trades reached S_{gain} over the random trades as a percentage.

It is clear that as the horizon increases, the number of trades decreases. This is clearly due to the lack of price data the further one looks back in time. Especially since the number of traded stocks on the JSE increased substantially in the late 1990s. In addition, it is fairly intuitive to see that bubbles which formed over long periods take longer to reform once they have burst, when compared to short-dated bubbles. One can have many 100 day bubbles in the space of 1500 days.

Table 4.1 shows that TS1 outperformed the random trading strategy for most of the horizons. Comparing column 3 to column 4 one can conclude that TS1 is hitting the stop gain much more frequently. This is especially true for the 200, 300, 600, 800 and 1300 day horizon. It is interesting to note that the percentage of random trades that reach S_{gain} is fairly consistent around 41%. Column 5 is important in that it better conveys the superiority of TS1 over the random results - it is not easy to quantify the difference between column 3 and 4. For a 200 and 300 day horizon, the number of trades that reach S_{gain} for TS1 is better than 91% (respectively 75%) of random trades. There is similar out-performance for the 600, 800, 1300 and 1500 day horizon.

The percentiles convey how poor the random strategy is in reaching S_{gain}. Even when looking at the 75th percentile the majority of percentages are below 55%. So most of the random trading strategies hit S_{loss}, rather than S_{gain}. One would also expect a smooth transition of success for TS1 from one horizon to the next, however this is not the case. There are large spikes as one moves down column 5, particularly when moving from 600, to 700, to 800 day horizon.

Tab. 4.1: Trading Strategy 1: Positive Bubble with $S_{\text{gain}} = S_{\text{loss}} = 5\%$

Horizon (Days)	No. Trades	% of $\bar{r}$ $= S_{\text{gain}}$ (TS1)	% of $\bar{r}$ $= S_{\text{gain}}$ (Rnd)	TS1 % above Rnd	25th Percentile (Rnd)	50th Percentile (Rnd)	75th Percentile (Rnd)
100	320	37.81%	41.84%	6.22%	40.00%	41.88%	43.75%
200	41	53.66%	41.93%	91.44%	36.59%	41.46%	46.34%
300	21	52.38%	41.85%	77.95%	33.33%	42.86%	47.62%
400	23	43.48%	42.00%	47.40%	34.78%	43.48%	47.83%
500	18	44.44%	41.63%	50.22%	33.33%	38.89%	50.00%
600	8	62.50%	42.14%	78.89%	25.00%	37.50%	50.00%
700	17	41.18%	41.94%	38.28%	35.29%	41.18%	47.06%
800	11	54.55%	41.84%	71.32%	27.27%	45.45%	54.55%
900	12	41.67%	41.83%	38.65%	33.33%	41.67%	50.00%
1000	17	41.18%	41.75%	38.96%	35.29%	41.18%	47.06%
1100	5	40.00%	41.93%	30.82%	20.00%	40.00%	60.00%
1200	13	46.15%	41.68%	52.42%	30.77%	38.46%	53.85%
1300	7	57.14%	41.99%	67.05%	28.57%	42.86%	57.14%
1400	14	42.86%	41.86%	42.81%	35.71%	42.86%	50.00%
1500	11	54.55%	41.93%	71.01%	27.27%	45.45%	54.55%

It is not entirely clear why the JLS model is successful over the 600 and 800 day horizons, but not over the 700 day horizon. Still it would seem that for certain time horizons, the stock data is presenting some LPPL structure which is being captured by the JLS model.

4.1.2 Negative Bubbles

Table 4.2 details the negative bubble calibration for $S_{\text{gain}} = S_{\text{loss}} = 7.5\%$. Again the number of trades falls as the horizon increases. The first noticeable characteristic of the table is that the values for column 3 and 4 are much higher than Table 4.1. This is because the Top40 index has been generally increasing since 2002, i.e. a trader would have done better by randomly buying than randomly short selling during the investment period. For instance the percentage of days between 3 June 2003 and 31 August 2015 that had a positive return was 54%, which is intuitively where the random strategy hovers around in column 4.

The difference between column 3 and 4 is much less pronounced in this table, and the reader will have to rely on column 5 and the percentiles in order to draw any conclusions. This is because (as seen in the percentiles) the random trading strategies are not distributed far from 54.8%. So doing just 3% better than the average random strategy will mean that you are already outperforming significantly. Once again the 200, 300 and 600 day horizon are producing better returns.

Tab. 4.2: Trading Strategy 1: Negative Bubble with $S_{\text{gain}} = S_{\text{loss}} = 2.5\%$.

Horizon (Days)	No. Trades	% of $\bar{r}$ $= S_{\text{gain}}$ (TS1)	% of $\bar{r}$ $= S_{\text{gain}}$ (Rnd)	TS1 % above Rnd	25th Percentile (Rnd)	50th Percentile (Rnd)	75th Percentile (Rnd)
100	691	49.64%	54.80%	0.24%	53.55%	54.85%	56.15%
200	370	55.68%	54.83%	60.57%	52.97%	54.86%	56.49%
300	275	58.55%	54.86%	87.81%	52.73%	54.91%	56.73%
400	353	49.86%	54.87%	2.31%	52.97%	54.96%	56.66%
500	335	53.73%	54.83%	32.21%	53.13%	54.93%	56.72%
600	361	58.45%	54.82%	91.13%	53.19%	54.85%	56.51%
700	263	51.71%	54.83%	13.89%	52.85%	54.75%	57.03%
800	291	50.86%	54.83%	7.81%	52.92%	54.98%	56.70%
900	189	57.14%	54.88%	70.94%	52.38%	55.03%	57.14%
1000	125	62.40%	54.86%	94.65%	52.00%	55.20%	57.60%
1100	64	53.13%	54.80%	34.33%	50.00%	54.69%	59.38%
1200	40	47.50%	54.84%	13.67%	50.00%	55.00%	60.00%
1300	50	42.00%	54.91%	2.71%	50.00%	54.00%	60.00%
1400	43	58.14%	54.88%	60.25%	48.84%	55.81%	60.47%
1500	64	51.56%	54.88%	25.43%	50.00%	54.69%	59.38%

Interestingly, 800 and 1300 day horizons are terrible achievers here, with 900 and 1400 day horizons taking their place. Again we see the unintuitive spikes as we move down column 5. Similar results are seen when increasing $S_{\text{gain}} = S_{\text{loss}}$ (see Appendix A.2) however the performance of the JLS model decreases the larger this value becomes. Interestingly we see consistent out-performance for the 1100 day horizon. Yet the previously successful horizons seem to chop and change depending on the value of S_{gain} and S_{loss}.

4.2 Trading Strategy 2

We remind the reader that for Trading Strategy 2 (TS2) the hypothetical trader short sells (buys) the underlying as soon as the JLS model indicates that it is in a positive (negative) bubble. The asset is then held until $t_c + t_{\text{plus}} \times \Delta$.

4.2.1 Positive Bubbles

Here we have actual returns for each trading strategy. Unfortunately it is evident that one would most likely have earned a negative return if they had followed one of these methods. However as mentioned previously, this dissertation seeks only to determine whether a JLS trading mechanism can outperform random trading. This is clearly evident in table 4.3. The mean TS2 return is consistently higher than the random return for all time horizons. Furthermore, its standard deviation is also consistently smaller and often times by a significant amount.

The most important piece of information is column 7. Here we see that TS2 performs better than roughly 90% of the 50 000 random trading strategies. Some horizons even push all the way up to 98%. The last column reveals how many of the TS2 trades from column 2 were improved upon by the random strategy as a percentage. Note that it is possible for many of the TS2 trades to be improved upon by changing the t_c date, without the overall return of the random strategy being larger. One may have many small improvements being offset by one or two large decreases. Importantly, the last column shows that the random strategy struggles to improve the return of TS2 for all horizons.

So it would seem that the JLS model is correctly selecting the t_c date, but the strategy is not trading the bubble in an optimal fashion - since the returns are mainly negative. In other words the JLS model is generating fairly accurate predictions of when the bubble will burst, but the trading strategy is not using this information wisely.

Tab. 4.3: Trading Strategy 2: Positive Bubble with $t_{plus} = 5\%$.

Horizon (Days)	No. Trades	Mean TS2 Return	Mean Rnd Return	Std of TS2 Return	Std of Rnd Return	% of TS2 Return > Rnd Return	% Improvement in Trades
100	78	-0.51%	-1.56%	3.37%	7.35%	89.93%	46.05%
200	54	-1.72%	-4.19%	6.49%	12.53%	94.48%	40.65%
300	64	-2.75%	-6.10%	9.48%	15.28%	96.98%	40.05%
400	54	-6.10%	-9.86%	12.73%	22.20%	91.55%	40.18%
500	66	-5.49%	-11.21%	11.48%	23.53%	98.73%	42.50%
600	29	-9.63%	-16.48%	17.91%	27.81%	93.63%	38.13%
700	33	-12.86%	-20.21%	18.43%	31.22%	93.64%	39.20%
800	13	-6.47%	-23.21%	14.74%	37.61%	96.62%	29.90%
900	27	-8.05%	-21.54%	16.43%	37.88%	98.61%	38.94%
1000	20	-4.53%	-20.04%	14.24%	41.15%	98.13%	36.57%
1100	5	-5.58%	-12.35%	14.83%	20.22%	76.46%	36.88%
1200	6	0.48%	-14.11%	11.85%	22.22%	94.01%	28.77%
1300	9	-4.89%	-28.42%	8.81%	39.18%	97.42%	21.43%
1400	5	0.54%	-17.71%	19.74%	21.07%	95.70%	20.80%
1500	5	-2.04%	-14.71%	14.32%	16.05%	88.62%	24.36%

4.2.2 Negative Bubbles

Table 4.4 is noticeable in that the majority of returns are positive with some values nearing 10%. The 42% average return seen for the 1100 day horizon is an outlier mainly because of the small number of trades made. It is hard to distinguish between an accurate model and pure luck when so few trades are made. However since we are looking at a 1100 day horizon, one would not expect there to be many bubble opportunities. Nevertheless, the general upward trend for the Top40 Index over the investment period is playing a major role in both the TS2 and random strategy returns being positive.

Contrary to the positive bubble scenario, TS2 fails to beat the random strategy for the majority of horizons. The second last column further supports this claim with the vast majority of TS2 returns being smaller than the random returns. The random trades are consistently improving the return for each individual trade, with many values in the last column hovering around 60%. It would seem that for negative bubbles, the JLS model is not producing credible predictions of the t_c date.

Tab. 4.4: Trading Strategy 2: Negative Bubble with $t_{plus} = 5\%$.

Horizon (Days)	No. Trades	Mean TS2 Return	Mean Rnd Return	Std of TS2 Return	Std of Rnd Return	% of TS2 Return > Rnd Return	% Improvement in Trades
100	316	1.01%	1.90%	7.95%	11.81%	8.72%	50.99%
200	128	2.71%	3.93%	12.58%	17.91%	22.45%	48.53%
300	31	0.24%	4.81%	12.88%	16.59%	5.94%	51.74%
400	13	-1.57%	7.74%	19.64%	19.95%	4.27%	57.90%
500	53	2.82%	10.78%	20.13%	29.55%	1.64%	54.54%
600	10	3.71%	12.29%	22.10%	28.99%	18.08%	56.86%
700	16	5.68%	15.60%	19.52%	40.83%	16.10%	50.29%
800	18	8.37%	14.55%	23.71%	37.39%	25.49%	54.65%
900	11	2.35%	9.44%	16.62%	21.75%	13.87%	61.92%
1000	13	8.56%	17.33%	33.25%	52.62%	30.57%	56.43%
1100	5	41.98%	19.24%	32.21%	48.91%	85.88%	26.57%
1200	7	-0.82%	15.66%	37.43%	49.53%	21.60%	53.08%
1300	7	-6.45%	12.16%	23.28%	40.22%	11.55%	64.75%
1400	8	-6.10%	12.18%	22.43%	35.47%	7.29%	61.21%
1500	7	5.62%	13.28%	24.53%	37.05%	31.78%	53.69%

Chapter 5

Conclusion

Financial markets are comprised of a vast number of heterogeneous agents, making the task of modelling their aggregate behaviour particularly difficult. The JLS model has made a courageous attempt to do just this, producing results that would indicate the presence of some LPPL structure in the JSE Top40 Index.

It is evident that the model is able to determine whether a stock is in a positive bubble, with the prediction of the crash date being quite accurate as seen in TS2. However this is highly dependent on the length of past price data used when calibrating the JLS model. The 200 and 300 day horizons display consistent out-performance, with the success of other horizons being dependent on the value of S_{gain} and S_{loss}. This highlights the notion that one cannot know a priori over what horizon a bubble is likely to form.

Calibration to negative bubbles enjoyed similar success, however this was more specific to the trading strategy used. There were definite horizons where TS1 significantly out-performed the random strategy, which did not continue when applying TS2. One may conclude that the JLS model is able to determine which stocks are in a negative bubble, with the prediction of the actual date of rebound being inaccurate, as seen in table 4.4.

The relative weaker performance for the JLS model and negative bubbles may come down to the stylized facts discussed in Section 2.2. These bounds were developed by calibrating the JLS model to a number of positive bubbles – not negative bubbles. It may be that the dynamics of LPPL rebounds are different to LPPL crashes and that there is an inherent flaw in the vetting procedure for negative bubbles. It is entirely possible for instance, that the number of oscillations in a LPPL rebound is far smaller. It would be interesting to see further research in determining similar parameter bounds for negative bubbles.

These stylized facts may also play a role in the turbulent success when calibrating each 100 extra days into the past. For instance, the bounds on m and ω were determined from bubbles that occurred over many years, and it may be the case that bubbles over different horizons have different LPPL structures. One hundred day bubbles may be more super-exponential and less oscillatory compared to long-dated bubbles. This would impact the search bounds for the parameters m and ω in equation (2.10).

In this respect it would be interesting for further research to develop more accurately the notion of there being stylized facts for the JLS model. It would seem that the bounds are not a one size fits all – especially for negative bubbles.

From a purely trading for profit perspective, it would be harsh to deem the JLS model unsuccessful - even though a large portion of the TS2 returns are negative. The model makes judgements based solely on a stock's price trajectory. There is no fundamental data taken into account such as price earnings, revenue, GDP, inflation, etc. The results shown above suggest that the JLS model would make a great indicator, amongst many other indicators, for attempting to profit off the prediction of stock crashes and rebounds.

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Appendix A

Strategy 1: Further Results

A.1 Positive Bubbles

Tab. A.1: Trading Strategy 1: Positive Bubble with $S_{\text{gain}} = S_{\text{loss}} = 2.5\%$.

Horizon (Days)	No. Trades	% of $\bar{r}$ $= S_{\text{gain}}$ (TS1)	% of $\bar{r}$ $= S_{\text{gain}}$ (Rnd)	TS1 % above Rnd	25th Percentile (Rnd)	50th Percentile (Rnd)	75th Percentile (Rnd)
100	178	44.94%	45.15%	44.91%	42.70%	44.94%	47.75%
200	45	64.44%	44.99%	99.45%	40.00%	44.44%	48.89%
300	66	39.39%	44.98%	15.55%	40.91%	45.45%	48.48%
400	26	53.85%	45.23%	75.85%	38.46%	46.15%	50.00%
500	23	34.78%	45.16%	11.22%	39.13%	43.48%	52.17%
600	14	78.57%	45.19%	98.86%	35.71%	42.86%	57.14%
700	65	47.69%	45.14%	61.23%	41.54%	44.62%	49.23%
800	16	68.75%	45.12%	94.82%	37.50%	43.75%	56.25%
900	17	47.06%	45.09%	46.97%	35.29%	47.06%	52.94%
1100	35	54.29%	45.27%	81.71%	40.00%	45.71%	51.43%
1200	23	47.83%	45.03%	52.56%	39.13%	43.48%	52.17%
1300	25	48.00%	45.26%	53.15%	40.00%	44.00%	52.00%
1400	14	50.00%	45.27%	53.92%	35.71%	42.86%	57.14%
1500	17	52.94%	45.36%	65.03%	35.29%	47.06%	52.94%
1600	8	75.00%	45.16%	91.05%	37.50%	50.00%	62.50%

Tab. A.2: Trading Strategy 1: Positive Bubble with $S_{\text{gain}} = S_{\text{loss}} = 7.5\%$.

Horizon (Days)	No. Trades	% of $\bar{r}$ $= S_{\text{gain}}$ (TS1)	% of $\bar{r}$ $= S_{\text{gain}}$ (Rnd)	TS1 % above Rnd	25th Percentile (Rnd)	50th Percentile (Rnd)	75th Percentile (Rnd)
100	292	32.88%	37.71%	3.59%	35.62%	37.67%	39.73%
200	36	47.22%	37.64%	84.28%	33.33%	38.89%	44.44%
300	17	47.06%	37.63%	71.91%	29.41%	35.29%	47.06%
400	20	35.00%	37.61%	32.48%	30.00%	35.00%	45.00%
500	14	35.71%	37.55%	34.88%	28.57%	35.71%	42.86%
600	8	37.50%	37.42%	37.24%	25.00%	37.50%	50.00%
700	28	39.29%	37.86%	49.24%	32.14%	39.29%	42.86%
800	16	56.25%	37.74%	89.36%	31.25%	37.50%	43.75%
900	18	33.33%	37.75%	27.12%	27.78%	38.89%	44.44%
1100	20	45.00%	37.80%	67.34%	30.00%	40.00%	45.00%
1200	9	55.56%	37.74%	77.52%	22.22%	33.33%	44.44%
1300	11	36.36%	37.81%	34.93%	27.27%	36.36%	45.45%
1400	7	42.86%	37.93%	47.00%	28.57%	42.86%	42.86%
1500	13	38.46%	37.61%	42.40%	30.77%	38.46%	46.15%
1600	7	28.57%	37.65%	19.02%	28.57%	42.86%	42.86%

Tab. A.3: Trading Strategy 1: Positive Bubble with $S_{\text{gain}} = S_{\text{loss}} = 10\%$.

Horizon (Days)	No. Trades	% of $\bar{r}$ $= S_{\text{gain}}$ (TS1)	% of $\bar{r}$ $= S_{\text{gain}}$ (Rnd)	TS1 % above Rnd	25th Percentile (Rnd)	50th Percentile (Rnd)	75th Percentile (Rnd)
100	142	28.87%	34.43%	6.66%	31.69%	34.51%	37.32%
200	33	45.45%	34.39%	87.63%	27.27%	33.33%	39.39%
300	16	43.75%	34.43%	71.12%	25.00%	31.25%	43.75%
400	19	31.58%	34.67%	31.23%	26.32%	36.84%	42.11%
500	36	25.00%	34.57%	8.23%	27.78%	33.33%	38.89%
600	24	37.50%	34.48%	54.88%	29.17%	33.33%	41.67%
700	25	36.00%	34.52%	48.22%	28.00%	36.00%	40.00%
800	16	37.50%	34.40%	50.93%	25.00%	31.25%	43.75%
900	14	28.57%	34.35%	23.31%	28.57%	35.71%	42.86%
1000	16	31.25%	34.67%	29.44%	25.00%	37.50%	43.75%
1100	18	27.78%	34.40%	20.42%	27.78%	33.33%	44.44%
1200	17	23.53%	34.52%	11.13%	29.41%	35.29%	41.18%
1300	7	42.86%	34.69%	54.10%	28.57%	28.57%	42.86%
1400	12	41.67%	34.39%	60.20%	25.00%	33.33%	41.67%
1500	6	16.67%	34.42%	8.01%	16.67%	33.33%	50.00%

A.2 Negative Bubbles

Tab. A.4: Trading Strategy 1: Negative Bubble with $S_{\text{gain}} = S_{\text{loss}} = 5\%$.

Horizon (Days)	No. Trades	% of $\bar{r}$ $= S_{\text{gain}}$ (TS1)	% of $\bar{r}$ $= S_{\text{gain}}$ (Rnd)	TS1 % above Rnd	25th Percentile (Rnd)	50th Percentile (Rnd)	75th Percentile (Rnd)
100	454	55.73%	58.15%	13.78%	56.61%	58.15%	59.69%
200	224	55.36%	58.19%	17.42%	55.80%	58.04%	60.27%
300	164	58.54%	58.21%	49.77%	55.49%	58.54%	60.98%
400	198	53.54%	58.16%	8.24%	55.56%	58.08%	60.61%
500	209	52.15%	58.16%	3.36%	55.98%	58.37%	60.29%
600	177	59.89%	58.17%	64.83%	55.37%	58.19%	60.45%
700	141	53.90%	58.19%	13.41%	55.32%	58.16%	60.99%
800	142	53.52%	58.19%	11.39%	55.63%	58.45%	61.27%
900	94	57.45%	58.16%	40.22%	54.26%	58.51%	61.70%
1000	59	62.71%	58.21%	71.15%	54.24%	57.63%	62.71%
1100	31	64.52%	58.35%	69.63%	51.61%	58.06%	64.52%
1200	17	64.71%	58.28%	61.06%	52.94%	58.82%	64.71%
1300	32	50.00%	58.11%	13.80%	53.13%	59.38%	65.63%
1400	22	68.18%	58.14%	76.85%	50.00%	59.09%	63.64%
1500	43	53.49%	58.27%	21.74%	53.49%	58.14%	62.79%

Tab. A.5: Trading Strategy 1: Negative Bubble with $S_{\text{gain}} = S_{\text{loss}} = 7.5\%$.

Horizon (Days)	No. Trades	% of $\bar{r}$ $= S_{\text{gain}}$ (TS1)	% of $\bar{r}$ $= S_{\text{gain}}$ (Rnd)	TS1 % above Rnd	25th Percentile (Rnd)	50th Percentile (Rnd)	75th Percentile (Rnd)
100	366	60.66%	62.27%	24.59%	60.66%	62.30%	63.93%
200	155	59.35%	62.24%	20.36%	59.35%	61.94%	64.52%
300	110	61.82%	62.26%	42.39%	59.09%	62.73%	65.45%
400	132	52.27%	62.35%	0.58%	59.09%	62.12%	65.15%
500	123	56.91%	62.38%	9.55%	59.35%	62.60%	65.04%
600	130	60.77%	62.27%	32.79%	59.23%	62.31%	65.38%
700	84	53.57%	62.35%	3.74%	58.33%	61.90%	65.48%
800	86	56.98%	62.25%	13.59%	58.14%	62.79%	66.28%
900	68	54.41%	62.34%	7.19%	58.82%	61.76%	66.18%
1000	42	64.29%	62.28%	53.95%	57.14%	61.90%	66.67%
1100	15	80.00%	62.32%	87.66%	53.33%	60.00%	73.33%
1200	12	50.00%	62.24%	11.76%	50.00%	66.67%	75.00%
1300	19	36.84%	62.21%	0.62%	52.63%	63.16%	68.42%
1400	15	73.33%	62.35%	72.48%	53.33%	60.00%	73.33%
1500	28	57.14%	62.34%	21.92%	57.14%	60.71%	67.86%

Tab. A.6: Trading Strategy 1: Negative Bubble with $S_{\text{gain}} = S_{\text{loss}} = 10\%$.

Horizon (Days)	No. Trades	% of $\bar{r}$ $= S_{\text{gain}}$ (TS1)	% of $\bar{r}$ $= S_{\text{gain}}$ (Rnd)	Model % above Rnd	25th Percentile (Rnd)	50th Percentile (Rnd)	75th Percentile (Rnd)
100	309	61.17%	65.53%	5.11%	63.75%	65.70%	67.31%
200	124	61.29%	65.55%	14.55%	62.90%	65.32%	68.55%
300	128	67.19%	65.55%	61.01%	62.50%	65.63%	68.75%
400	96	56.25%	65.53%	2.34%	62.50%	65.63%	68.75%
500	100	60.00%	65.52%	10.17%	62.00%	66.00%	69.00%
600	99	64.65%	65.50%	37.76%	62.63%	65.66%	68.69%
700	62	61.29%	65.47%	20.18%	61.29%	66.13%	69.35%
800	67	61.19%	65.45%	19.31%	61.19%	65.67%	70.15%
900	44	61.36%	65.50%	23.18%	61.36%	65.91%	70.45%
1000	25	72.00%	65.60%	66.84%	60.00%	64.00%	72.00%
1100	13	76.92%	65.78%	70.29%	53.85%	69.23%	76.92%
1200	6	50.00%	65.66%	11.14%	50.00%	66.67%	83.33%
1300	9	22.22%	65.70%	0.15%	55.56%	66.67%	77.78%
1400	10	60.00%	65.62%	23.54%	60.00%	70.00%	80.00%
1500	17	47.06%	65.69%	3.27%	58.82%	64.71%	70.59%

Appendix B

Strategy 2: Further Results

B.1 Positive Bubbles

Tab. B.1: Trading Strategy 2: Positive Bubble with $t_{plus} = 0\%$.

Horizon (Days)	No. Trades	Mean TS2 Return	Mean Rnd Return	Std of TS2 Return	Std of Rnd Return	% of TS2 Return > Rnd Return	% Improvement in Trades
100	316	-0.56%	-1.16%	3.93%	7.22%	93.55%	48.04%
200	319	-1.59%	-2.30%	7.71%	11.61%	87.21%	46.49%
300	12	-2.74%	-5.73%	11.19%	13.75%	79.91%	36.35%
400	24	-2.49%	-5.35%	6.68%	13.78%	88.09%	39.20%
500	108	-1.72%	-4.99%	8.49%	16.26%	99.46%	44.05%
600	51	-4.15%	-7.78%	11.40%	19.83%	93.77%	44.48%
700	54	-5.79%	-9.97%	11.32%	22.54%	95.04%	41.29%
800	62	-5.37%	-10.83%	15.71%	27.16%	97.70%	44.05%
900	68	-1.93%	-6.38%	8.36%	21.22%	98.56%	47.33%
1000	45	-2.19%	-3.97%	7.04%	17.05%	77.85%	46.40%
1100	48	-0.93%	-3.80%	6.03%	16.20%	92.18%	39.65%
1200	11	2.77%	-5.03%	5.30%	15.86%	96.59%	32.88%
1300	31	-5.61%	-11.05%	10.82%	27.57%	91.53%	38.86%
1400	78	-4.32%	-5.96%	19.84%	23.02%	77.27%	47.31%
1500	51	-6.88%	-7.01%	19.66%	23.80%	50.71%	44.74%

Tab. B.2: Trading Strategy 2: Positive Bubble with $t_{plus} = 10\%$.

Horizon (Days)	No. Trades	Mean TS2 Return	Mean Rnd Return	Std of TS2 Return	Std of Rnd Return	% of TS2 Return > Rnd Return	% Improvement in Trades
100	73	-0.64%	-2.12%	3.69%	8.65%	93.22%	44.10%
200	48	-1.80%	-4.84%	5.83%	13.21%	94.86%	39.96%
300	54	-3.12%	-8.05%	10.04%	17.70%	98.51%	37.54%
400	47	-8.56%	-13.29%	13.12%	25.81%	91.54%	40.37%
500	55	-7.52%	-15.68%	15.81%	28.39%	99.21%	40.62%
600	48	-9.49%	-19.59%	17.44%	33.57%	99.41%	39.68%
700	30	-14.23%	-26.79%	23.20%	36.80%	98.29%	34.49%
800	49	-16.16%	-31.67%	26.21%	50.92%	99.49%	39.07%
900	55	-16.42%	-35.44%	28.90%	59.51%	99.87%	37.44%
1000	15	-3.83%	-29.39%	18.66%	48.34%	99.39%	29.69%
1100	8	-5.89%	-23.34%	11.62%	30.09%	94.49%	28.23%
1200	14	-16.78%	-33.78%	19.45%	43.18%	94.47%	34.73%
1300	7	-20.20%	-51.34%	27.56%	56.81%	95.27%	26.67%
1400	12	-18.18%	-38.83%	22.81%	50.54%	95.10%	34.68%
1500	15	-48.79%	-55.80%	50.71%	70.62%	64.13%	43.61%

Tab. B.3: Trading Strategy 2: Positive Bubble with $t_{plus} = 15\%$.

Horizon (Days)	No. Trades	Mean TS2 Return	Mean Rnd Return	Std of TS2 Return	Std of Rnd Return	% of TS2 Return > Rnd Return	% Improvement in Trades
100	240	-1.46%	-2.99%	5.22%	11.49%	98.18%	44.49%
200	191	-4.03%	-6.93%	11.75%	20.81%	98.32%	41.56%
300	8	-1.24%	-12.65%	16.61%	20.41%	94.83%	29.01%
400	15	-5.73%	-13.73%	12.49%	21.37%	94.17%	29.05%
500	6	-15.11%	-21.10%	19.45%	22.71%	75.58%	39.31%
600	45	-11.98%	-23.56%	24.31%	37.48%	99.37%	37.54%
700	23	-18.58%	-31.70%	26.53%	39.69%	96.19%	34.32%
800	10	-12.50%	-38.75%	21.03%	48.20%	97.32%	27.94%
900	16	-18.87%	-41.60%	27.90%	51.31%	97.74%	34.54%
1000	20	-22.93%	-45.15%	61.31%	66.53%	95.92%	36.85%
1100	46	-27.73%	-50.04%	48.88%	77.10%	99.33%	38.37%
1200	44	-31.01%	-56.10%	60.93%	88.79%	99.21%	36.25%
1300	31	-30.86%	-63.08%	61.17%	93.10%	99.40%	34.55%
1400	36	-37.60%	-62.58%	51.87%	90.86%	97.95%	40.80%
1500	35	-36.80%	-69.77%	42.81%	91.01%	99.68%	36.88%

B.2 Negative Bubbles

Tab. B.4: Trading Strategy 2: Negative Bubble with $t_{plus} = 0\%$.

Horizon (Days)	No. Trades	Mean TS2 Return	Mean Rnd Return	Std of TS2 Return	Std of Rnd Return	% of TS2 Return > Rnd Return	% Improvement in Trades
100	371	0.70%	0.99%	5.38%	8.42%	25.33%	50.45%
200	81	0.94%	2.01%	7.03%	11.25%	19.11%	51.35%
300	69	-0.16%	1.63%	6.89%	9.74%	5.82%	48.58%
400	33	-0.07%	2.56%	11.40%	11.66%	8.64%	48.13%
500	167	0.21%	2.71%	10.78%	15.10%	0.85%	50.66%
600	19	2.30%	5.43%	12.28%	19.53%	24.33%	46.38%
700	49	-0.21%	3.76%	10.58%	18.57%	4.55%	55.62%
800	75	-0.74%	1.95%	8.77%	13.67%	3.23%	51.07%
900	85	-0.05%	0.67%	4.54%	6.01%	12.21%	52.73%
1000	17	4.00%	4.43%	20.96%	20.00%	50.48%	49.39%
1100	5	18.31%	11.21%	25.12%	26.86%	74.86%	42.73%
1200	29	-0.94%	3.03%	10.30%	22.37%	19.13%	44.21%
1300	53	-0.88%	0.73%	5.08%	10.27%	13.34%	55.64%
1400	27	-1.46%	0.70%	6.44%	7.83%	7.58%	54.53%
1500	146	0.53%	0.48%	3.39%	7.58%	55.27%	48.69%

Tab. B.5: Trading Strategy 2: Negative Bubble with $t_{plus} = 10\%$.

Horizon (Days)	No. Trades	Mean TS2 Return	Mean Rnd Return	Std of TS2 Return	Std of Rnd Return	% of TS2 Return > Rnd Return	% Improvement in Trades
100	83	1.91%	2.42%	7.24%	11.39%	34.51%	50.33%
200	40	3.12%	5.77%	9.96%	19.62%	19.97%	52.13%
300	7	4.70%	7.26%	8.79%	20.23%	41.53%	47.60%
400	35	-2.72%	10.10%	19.20%	29.05%	0.23%	61.98%
500	43	3.89%	14.88%	23.95%	35.29%	1.44%	55.37%
600	26	2.38%	19.06%	22.82%	42.06%	1.35%	60.86%
700	25	12.72%	24.55%	32.60%	57.78%	14.79%	51.34%
800	12	12.03%	25.65%	23.15%	54.01%	20.99%	55.31%
900	8	7.95%	16.55%	21.65%	29.21%	20.72%	58.76%
1000	6	10.07%	20.18%	43.71%	42.79%	30.99%	59.98%
1100	5	39.49%	28.96%	53.14%	51.68%	67.30%	24.63%
1200	5	1.65%	37.56%	45.42%	88.15%	28.89%	48.73%
1300	5	-17.66%	28.24%	32.74%	67.27%	4.96%	68.64%
1400	6	-14.32%	34.43%	29.01%	73.25%	3.25%	68.65%
1500	5	9.52%	27.11%	29.51%	58.93%	31.12%	54.94%

Tab. B.6: Trading Strategy 2: Negative Bubble with $t_{plus} = 15\%$.

Horizon (Days)	No. Trades	Mean TS2 Return	Mean Rnd Return	Std of TS2 Return	Std of Rnd Return	% of TS2 Return > Rnd Return	% Improvement in Trades
100	78	2.07%	2.84%	8.35%	12.39%	29.53%	50.37%
200	38	3.35%	7.59%	9.82%	24.48%	13.65%	54.79%
300	7	7.36%	8.70%	8.07%	22.73%	49.64%	47.02%
400	33	-0.18%	12.33%	24.11%	32.03%	0.79%	61.12%
500	40	2.60%	17.90%	26.46%	39.85%	0.39%	59.40%
600	24	3.82%	23.39%	24.46%	47.83%	1.30%	61.29%
700	21	13.45%	28.71%	36.28%	63.19%	12.89%	52.47%
800	11	14.94%	31.27%	22.72%	60.70%	20.81%	54.77%
900	7	10.95%	21.49%	24.49%	32.26%	20.65%	57.83%
1000	5	16.72%	27.42%	49.65%	48.92%	34.67%	57.29%
1100	5	46.04%	34.35%	56.19%	59.42%	67.29%	23.90%
1200	5	8.33%	42.29%	71.71%	85.67%	35.26%	50.56%
1300	5	-11.30%	35.70%	37.27%	78.56%	13.09%	64.27%
1400	5	-17.18%	62.07%	36.98%	111.20%	5.07%	72.13%
1500	5	-7.54%	49.72%	19.88%	93.62%	8.11%	67.27%